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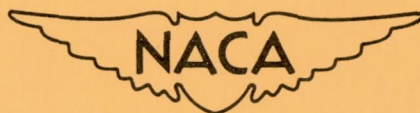
# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4027

APPLICATION OF OBLIQUELY MOUNTED STRAIN GAGE TO  
MEASUREMENT OF RESIDUAL STRESSES IN DISKS

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SUMMARY

A simple method is presented whereby the residual stress distribution in a disk can be determined with the aid of the obliquely mounted strain gage when the directions of the principal stresses are radial and tangential. The strain gages are mounted in such a way as to make them sensitive only to the tangential stresses in the disk. The disk is then cut so as to relieve completely the residual tangential stresses, and the strain gages therefore indicate directly the residual tangential stresses that were present in the disk. This procedure consists of making several preliminary radial saw cuts to reduce the over-all residual stress level and then one final diametral saw cut adjacent to the mounted strain gages through the plane where the residual tangential stress is desired. The residual radial stress distribution is calculated from the measured tangential stresses by using the equilibrium equation for a disk.

Results of measurements on overspeeded disks are presented and compared to the calculated values. The average deviation between measured and calculated values of residual tangential stress for two disks was approximately 1000 pounds per square inch when the recommended cutting procedure was followed.

INTRODUCTION

A great many instances arise when it would be of value to know the magnitude of the residual stresses in a disk. Many turbine disk failures such as those initiated by rim cracking have been attributed to residual stresses, and the first step in solving this problem would be to measure these stresses. A knowledge of the magnitude and location of residual stresses could greatly aid the designers and manufacturers in producing disks which would have longer lives.

There is at present a number of methods that can be used for measuring the residual stresses in a disk (ref. 1). The boring-out process and the isolation of small volumes with strain gages mounted on them are two of the more commonly used methods today. These, however, are costly, time consuming, and require a great deal of care to achieve accuracy.

This report describes a method of adapting a strain gage (ref. 2) to the direct measurement of the residual stresses in a disk. The process involves mounting a series of strain gages on the disk, cutting the disk in the manner recommended herein, and recording the changes in gage readings due to the relief of the residual tangential stress. This method is faster than those of reference 1 and gives results that compare favorably with values predicted by the deformation theory of plasticity.

### SYMBOLS

|                   |                                                                          |
|-------------------|--------------------------------------------------------------------------|
| $E$               | Young's modulus of elasticity                                            |
| $h$               | function expressing radial thickness variation in disk                   |
| $h_R$             | thickness of disk at radius $R$                                          |
| $R$               | radial coordinate measured from center of disk                           |
| $\epsilon_\theta$ | strain in $\theta$ -direction                                            |
| $\theta$          | gage mounting angle measured from direction of one of principal stresses |
| $\nu$             | Poisson's ratio                                                          |
| $\sigma_r$        | radial stress                                                            |
| $\sigma_{r,R}$    | radial stress at radius $R$                                              |
| $\sigma_t$        | tangential stress                                                        |

### THEORY OF STRESS GAGE

When the directions of the principal residual stresses are known, it becomes possible to apply the strain gage to the measurement of those residual stresses. In most disk problems, the directions of the principal residual stresses are in the radial and tangential directions because of symmetry. The method to be presented is for disks having principal residual stresses in these radial and tangential directions.



Consider a specimen subjected to a uniaxial stress  $\sigma_r$ . If a strain gage is mounted on this specimen at various angles to the principal direction, the strain readings will be a function of the gage orientation and the magnitude of  $\sigma_r$  as indicated by the  $\sigma_r$  family of curves in figure 1. If the  $\sigma_r$  is removed and the specimen is subjected to a uniaxial stress  $\sigma_t$ , the strain readings will be a function again of the gage orientation and the magnitude of  $\sigma_t$  as indicated by the  $\sigma_t$  family of curves. If the specimen is now subjected to a biaxial stress state, the strain at any angle  $\theta$  will be the algebraic sum of the strains due to  $\sigma_r$  and  $\sigma_t$ . An example of this is shown in figure 1 for a biaxial stress state of  $\sigma_{t3}$  and  $\sigma_{r3}$  and a strain gage mounted at  $10^\circ$  from the  $\sigma_t$  direction. The strain at this  $10^\circ$  angle is A-B minus A-C. If  $\sigma_{t3}$  is changed in value to  $\sigma_{t2}$  and  $\sigma_{r3}$  is changed to  $\sigma_{r1}$ , the strain at this same  $10^\circ$  angle would now be A-D minus A-E. It can be seen, therefore, that by knowing the angle  $\theta$  and measuring the strain at  $\theta$ , the values of  $\sigma_t$  and  $\sigma_r$  cannot in general be determined since any number of combinations of  $\sigma_t$  and  $\sigma_r$  can produce the same strain at any one particular angle.

There is one way in which the stress value in a biaxial stress state can be obtained from a single strain gage. It will be noted in figure 1 that the  $\sigma_r$  family of curves all cross the zero strain line at the same angle. If a strain gage is mounted on a specimen at that particular angle, the strain reading would be related to the magnitude of  $\sigma_t$  and would not depend at all on the magnitude of  $\sigma_r$ . A gage mounted at that angle would be a stress gage because its strain reading would be directly proportional to the magnitude of  $\sigma_t$ .

The angle  $\theta$  at which the gage is to be mounted and the relation between the strain reading  $\epsilon_\theta$  at that angle  $\theta$  and the stress in the tangential direction  $\sigma_t$  are given in reference 2 as

$$\left. \begin{aligned} \theta &= \frac{1}{2} \cos^{-1} \frac{1 - \nu}{1 + \nu} \\ \sigma_t &= \frac{E}{1 - \nu} \epsilon_\theta \end{aligned} \right\} \quad (1)$$

where  $E$  is Young's modulus and  $\nu$  is Poisson's ratio for the disk material.

If a series of these stress gages is mounted adjacent to a diametral line of a disk and the disk is cut along this diameter, a free surface is produced along which the normal stress or, in this case, the tangential stress, must be zero. The stress gages, which are sensitive to



the tangential stress, will therefore indicate directly the complete change of tangential stress from the original residual value to zero. The change in strain reading  $\epsilon_\theta$  from any gage can be converted to a residual tangential stress at that gage location by the use of equation (1). Figure 2 shows how a series of gages might be laid out on a disk when the residual tangential stresses are to be measured.

In order to obtain the residual radial stress distribution, the equilibrium equation for a disk can be used. This equation:

$$\frac{d}{dR} (Rh\sigma_r) - h\sigma_t = 0 \quad (2)$$

when integrated gives the following equation for the solution of the residual radial stress at radius  $R$ ,  $\sigma_{r,R}$

$$\sigma_{r,R} = \frac{1}{Rh_R} \int_0^R h\sigma_t dR \quad (3)$$

where  $h$  is the disk thickness which may be a function of radius, and  $h_R$  is the disk thickness at  $R$ . With equation (3) it becomes a simple matter to calculate the residual radial stress distribution from the graphical integration of the measured residual tangential stress distribution and the known thickness variation of the disk. It should be noted that a stress gage indicates an average reading over the gage area. For this reason the accuracy of a gage reading will be lower in an area of a high stress gradient than in an area of a constant stress. This method of residual stress measurement is therefore limited to cases where the stress gradient is not too great across any one stress gage.

## VERIFICATION OF METHOD

### Experimental Procedure

A number of parallel-sided disks were machined from forged AMS-5602 steel blanks to a diameter of 12 inches and a thickness of  $3/4$  inch. The finished disks were then stress-relieved at  $1300^\circ\text{F}$  for 12 hours in an annealing compound and allowed to furnace-cool. In order to be sure that this treatment was relieving the residual stresses due to forging and machining, a disk was instrumented with stress gages and then cut. The measured residual stresses in this disk were found to be less than  $\pm 500$  psi. Residual stresses were induced in the disks by spinning them to 24,500 rpm. Tensile tests indicated that the 0.2-percent offset yield strength was 29,000 psi, and computations show that plastic flow started at about 16,000 rpm. The shaft attachment to the disks was made as small as possible in order to minimize its effect on the residual stress distribution. Figure 3 is a sketch of the disk and shaft attachment.



After spinning, the shafts were cut off flush to the disk surface, and 30 SR-4 type AB-11 strain gages were then carefully applied to the disks at  $27^\circ$  to the tangential direction, adjacent to a diametral line at locations where the residual stress was desired. The  $27^\circ$  was calculated from equation (1) for  $\nu = 0.26$ . A number of different procedures were used in cutting the various disks, and the procedure suggested for best results will be discussed in the section RESULTS AND DISCUSSION.

A set of gage readings was taken before the first cut and then after each cutting operation. The clamps holding the disk to the saw table were removed for each set of gage readings. Heating of the gages due to the cutting of the disk was minimized by using an excess of cutting lubricant supplied throughout the machining and data-taking process.

An automatic-feed band saw was used to make all the cuts, and with a cutting feed of  $1/2$  inch per minute a complete test could be made in approximately 2 hours. A photograph of a test is shown in figure 4.

#### Analytical Computations

In order to calculate the residual stress distribution in an overspeeded disk, it was necessary to determine the stress strain curve for the disk material. A number of tensile specimens were machined from a few disks where the specimens were taken in both the radial and tangential directions. Test results showed that this material was very nearly isotropic, and the average stress strain curve as determined by several tests is shown in figure 5.

The plastic stress distribution was calculated by the method of reference 3 using 24 equally spaced stations over the 6-inch radius. The elastic stress distribution was then calculated by the same method and number of stations but now by using the new disk dimensions and contour as calculated by the plasticity theory. The residual stress distribution is then obtained by taking the difference between the tangential plastic and tangential elastic stresses to obtain the tangential residual stress and the difference between the radial plastic and radial elastic stresses to obtain the radial residual stress. The results of these calculations can be seen in figures 6 and 7. Figure 6 also includes the calculated contour of the disk after plastic flow has occurred. This method of calculating these stress distributions is verified experimentally in reference 4, and some of the results of reference 4 are presented in figure 8.

For this particular problem a very large percentage variation in the calculated residual stresses could result from a very small percentage variation of the calculated plastic and elastic stresses. For example, at the disk center the plastic stress is about 70,000 psi and the elastic stress is about 77,000 psi, which results in a residual stress of about 7000 psi. If both the elastic and plastic stresses were in error by 1 percent, the residual stress could be in error by as much



as 21 percent. It therefore appears that for this particular problem a very high degree of accuracy is required in the calculation of the plastic and elastic stress distributions if the calculated residual stress distribution is to be used as a check on the experimentally determined residual stress distribution.

## RESULTS AND DISCUSSION

Cutting procedure had a marked influence on the residual stress measurements. When a disk was cut along only one diameter, the measured stresses were in poor agreement with the theoretical values, and there was neither the expected radial symmetry of stress nor the equilibrium of force across this diameter (as calculated by integrating the measured tangential stresses). These results seemed to indicate that some additional plastic flow had occurred in the region of the stress gages during the cutting operation.

To ensure that the stresses being measured are the residual elastic stresses in the undisturbed disk, a procedure for cutting the disk is desired in which additional plastic flow does not occur in the region of the stress gages. Cutting of the disk should proceed in such a way that the peak stresses in the region being measured are always reduced as the cutting proceeds. This procedure eliminates the possibility of any additional plastic flow occurring in the gage region and, hence, ensures the correct residual stress measurements.

The cutting procedure that was finally adopted is shown in figure 9. This procedure reduces the possibility of any plastic flow in the gage regions by taking three distinct precautions. The first precaution is to relieve the residual stresses in the disk as symmetrically (with respect to the gages) as possible. The second precaution is to relieve as much of the stresses as possible by cuts that are not near the gages. It was found that cuts near gages under stress affect the reading of those gages; the higher the stress the greater the effect. This may be due to the local stresses set up during the cutting operation. The final precaution exercised was to keep the distance of the final saw cut  $1/8$  inch from the active gage element (fig. 9). This distance actually depends on the method and severity of the cut and was found to be necessary because of the very local plastic deformations that might affect the gages and their bond to the disk.

In order to determine the relative accuracy of the method and procedure presented herein, a comparison was made between the experimental results and those calculated by the method of reference 3 (fig. 10). It should be noted that the two sets of experimental results shown differ slightly even though the same procedure was used on the two disks. Quite good agreement was obtained between theory and experiment. The average deviation between the measured and calculated values of residual



tangential stress was determined for each test disk by adding the absolute values of the differences between the measured and calculated stresses at each gage location and dividing by the number of gages. These calculations gave an average deviation of about 1000 psi for both test disks shown. This value is considered quite good for residual stress measurements.

A very important and independent check on the experimental results is the condition of equilibrium of force across a diameter of the disk. It should be noted from equation (3) that if the integral of the force over the area is zero, the radial stress (computed from the measured tangential stress) at the rim of the disk will also be zero. Figure 9(b) shows how close to zero the radial stress at the rim is, and this minor deviation from zero is an indication of the proximity to equilibrium.

#### CONCLUDING REMARKS

The obvious advantages of this method of determining the residual stress distribution in a disk are simplicity, maximum information for a minimum number of gages, and a minimum amount of testing time. The method and procedure outlined in this report can be used to obtain the residual stresses in any problem where the directions of principal stresses are known and an equilibrium equation exists relating the two principal stresses. The method therefore proves most valuable as a means of analyzing disks such as compressor or turbine rotor disks.

Lewis Flight Propulsion Laboratory  
National Advisory Committee for Aeronautics  
Cleveland, Ohio, May 14, 1957

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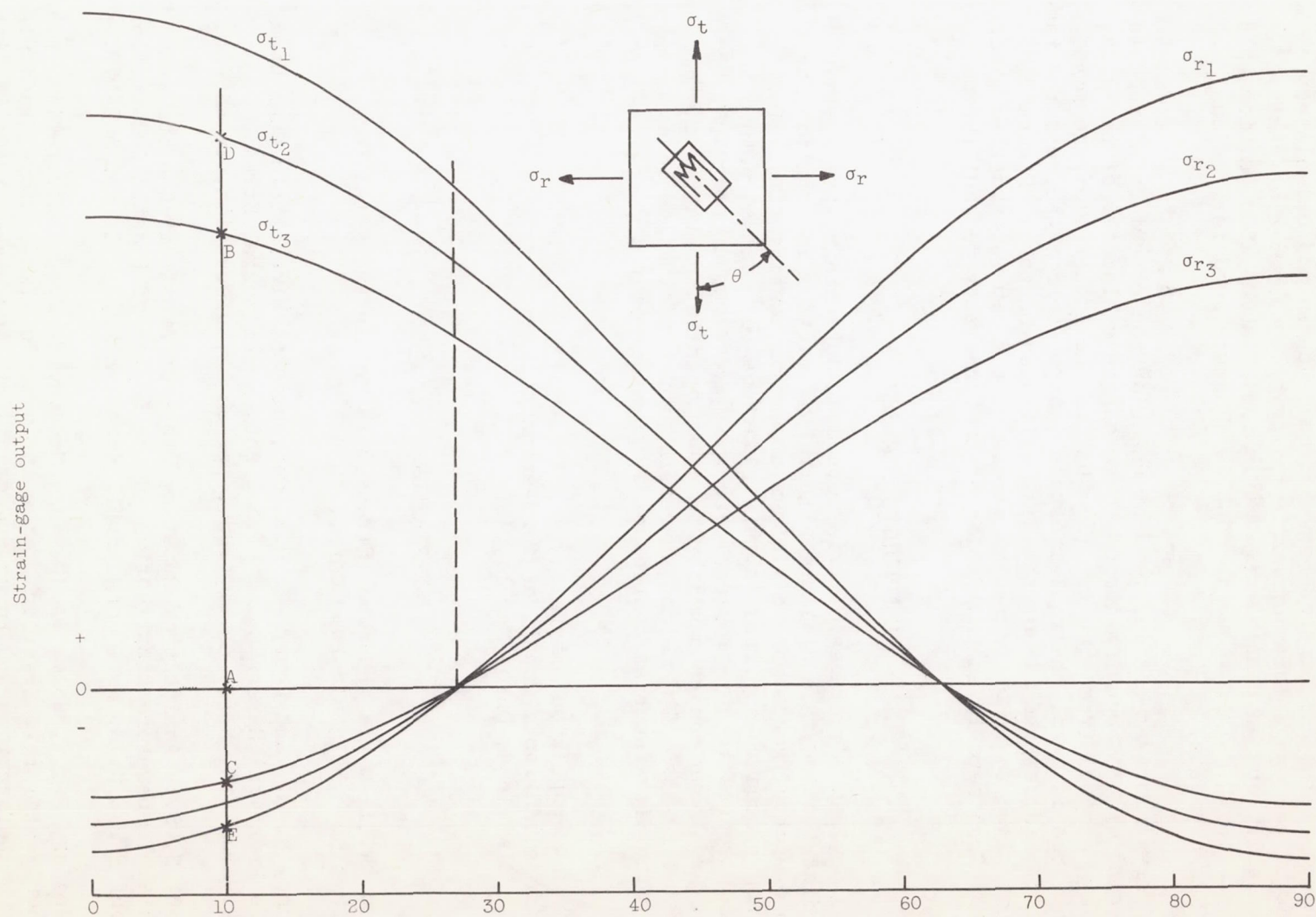
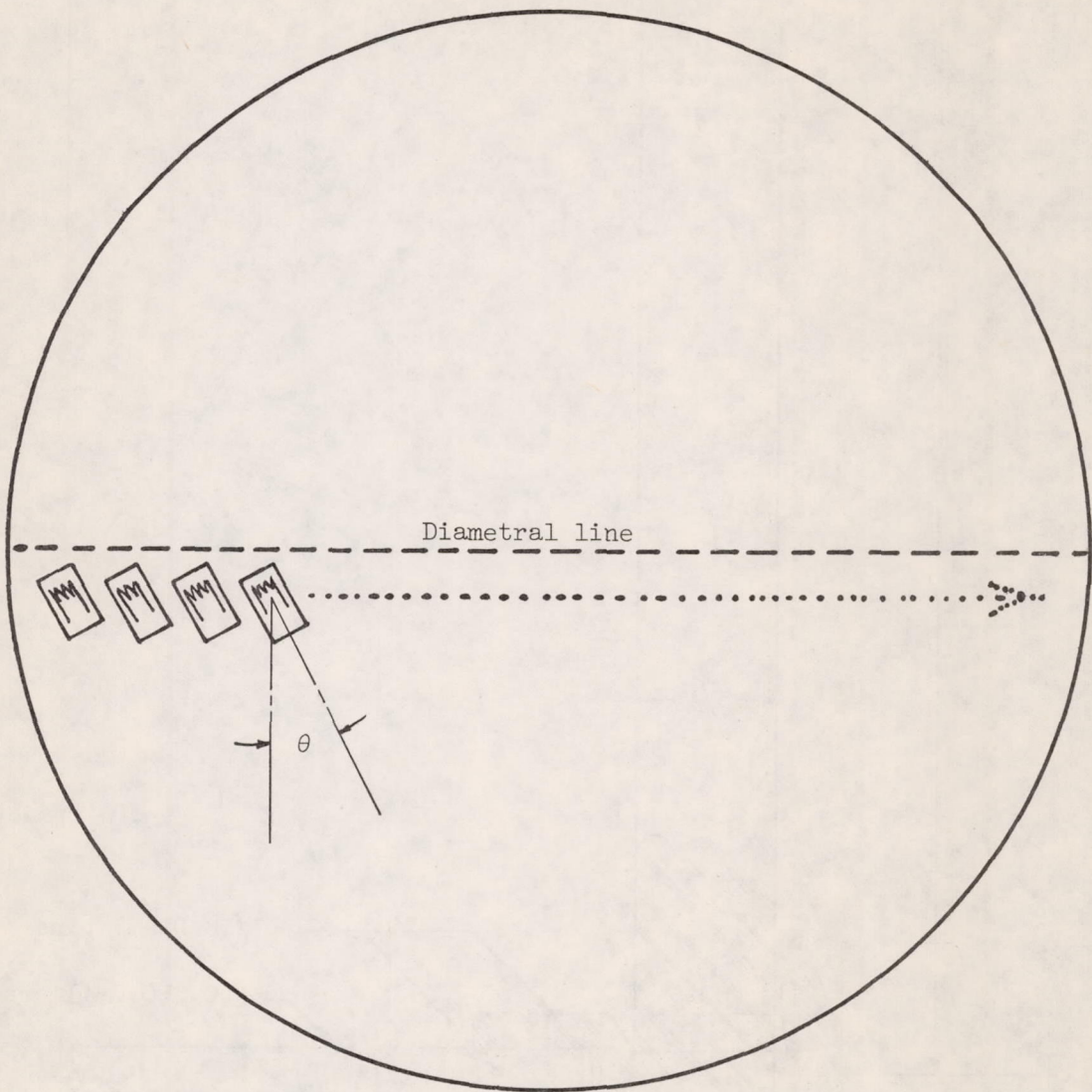


Figure 1. - Variation in strain-gage output with mounting angle for Poisson's ratio of 0.26.





$$\text{Gage mounting angle } \theta = \frac{1}{2} \cos^{-1} \left( \frac{1 - \nu}{1 + \nu} \right), \text{ deg}$$

Figure 2. - Layout of stress gages for measurement of residual tangential stresses in a disk.

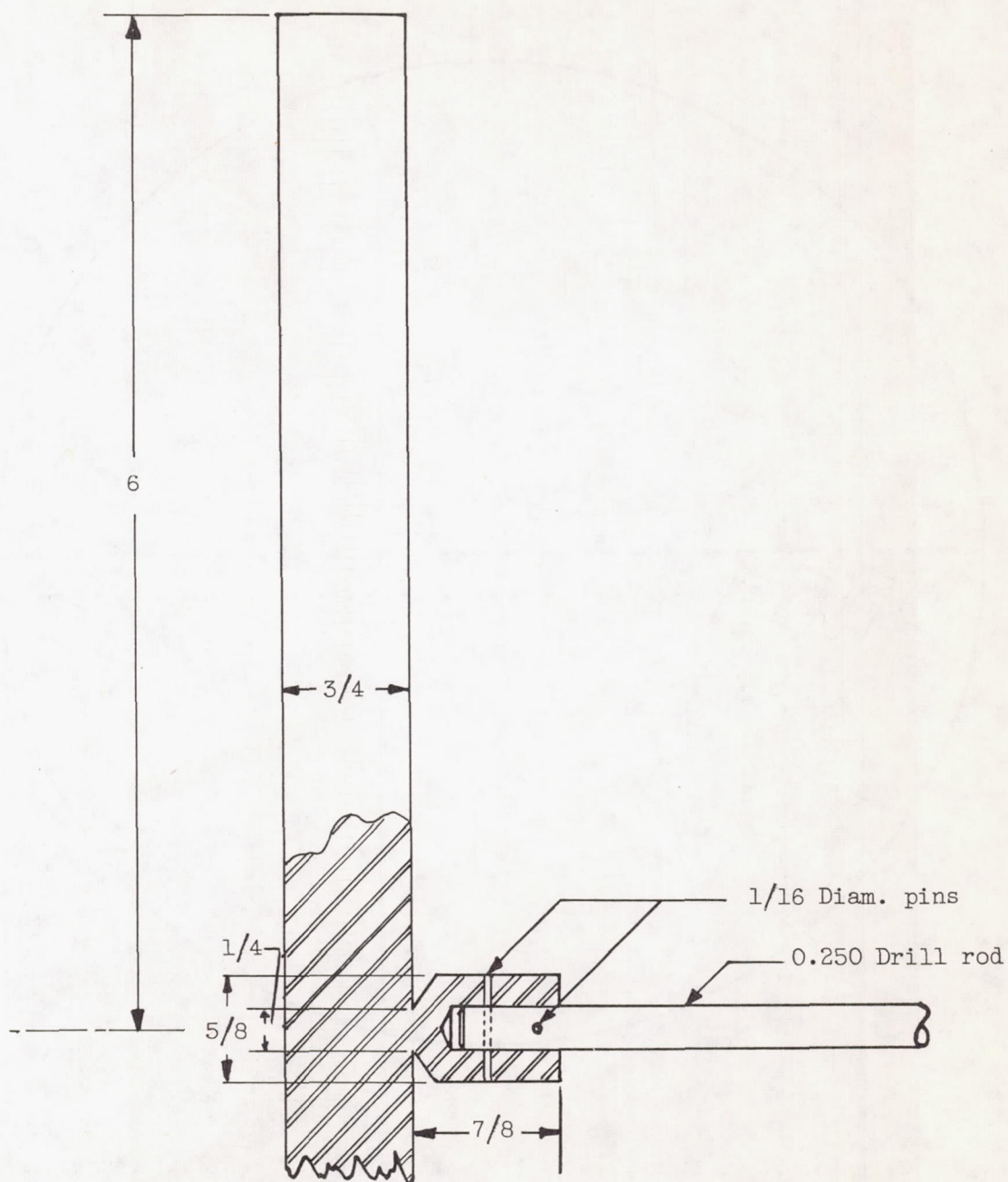


Figure 3. - Test disk. (All dimensions in inches.)



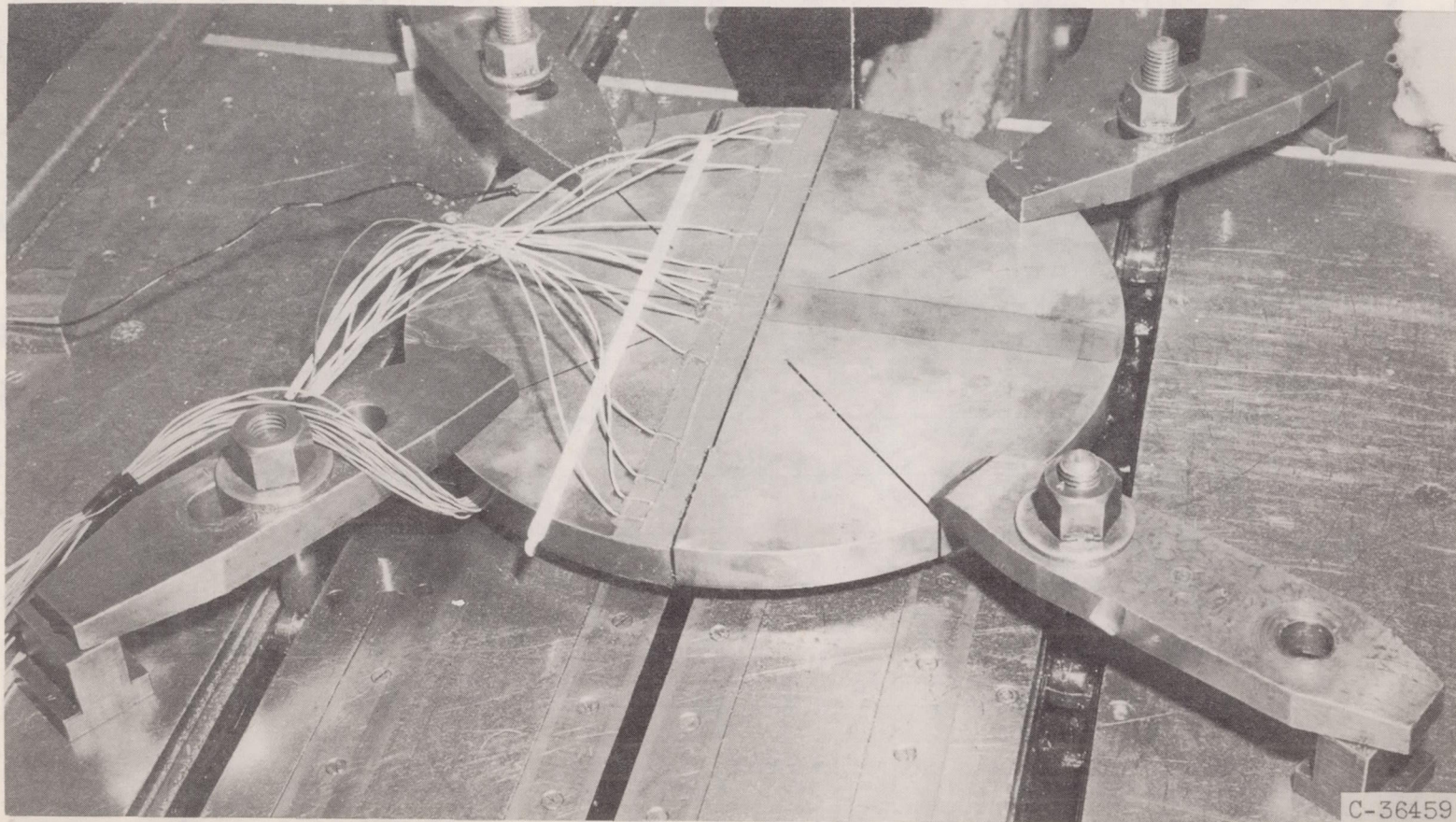


Figure 4. - Test disk after overspeeding and cutting.



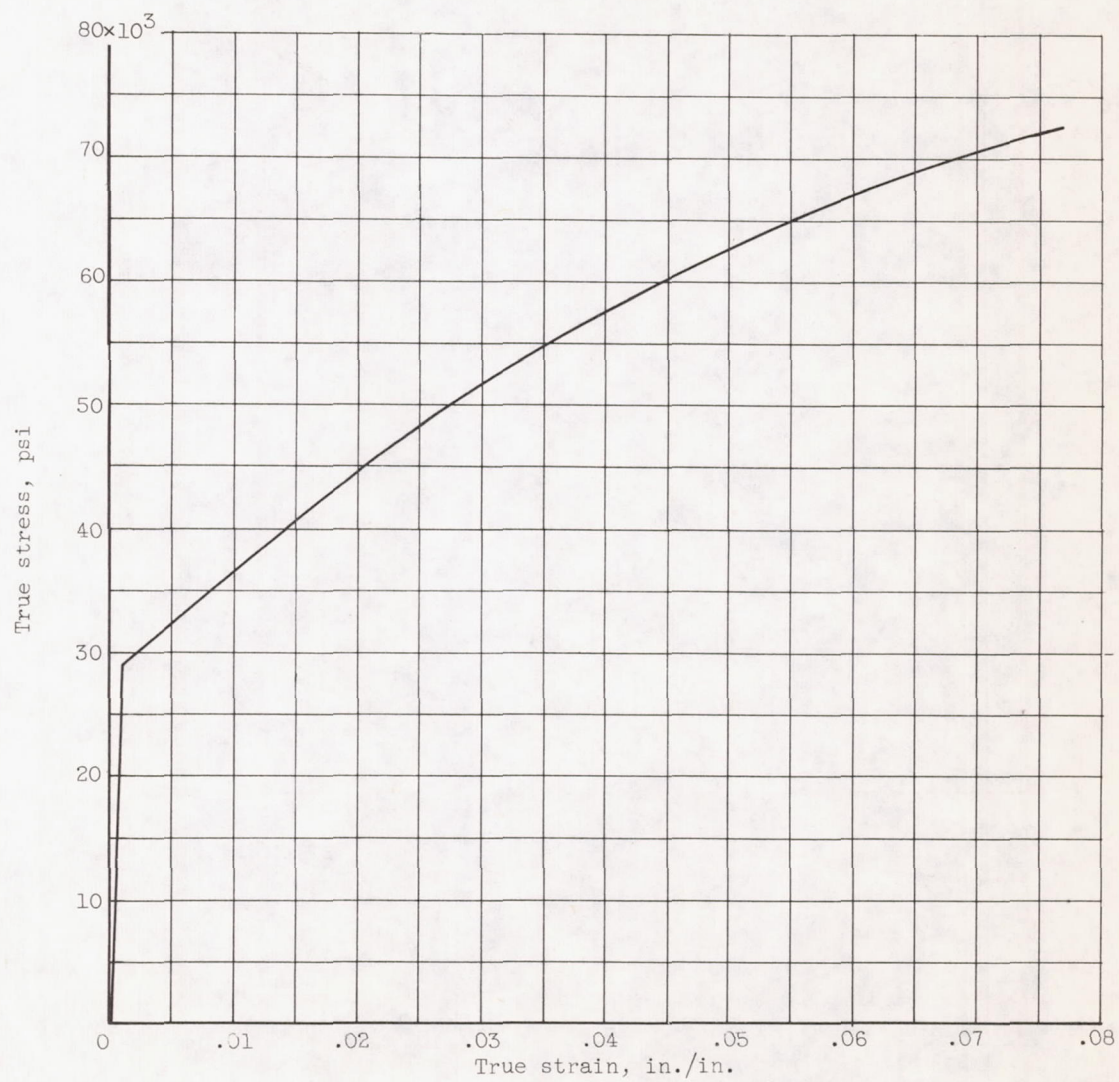


Figure 5. - Stress strain curve for AMS-5602.



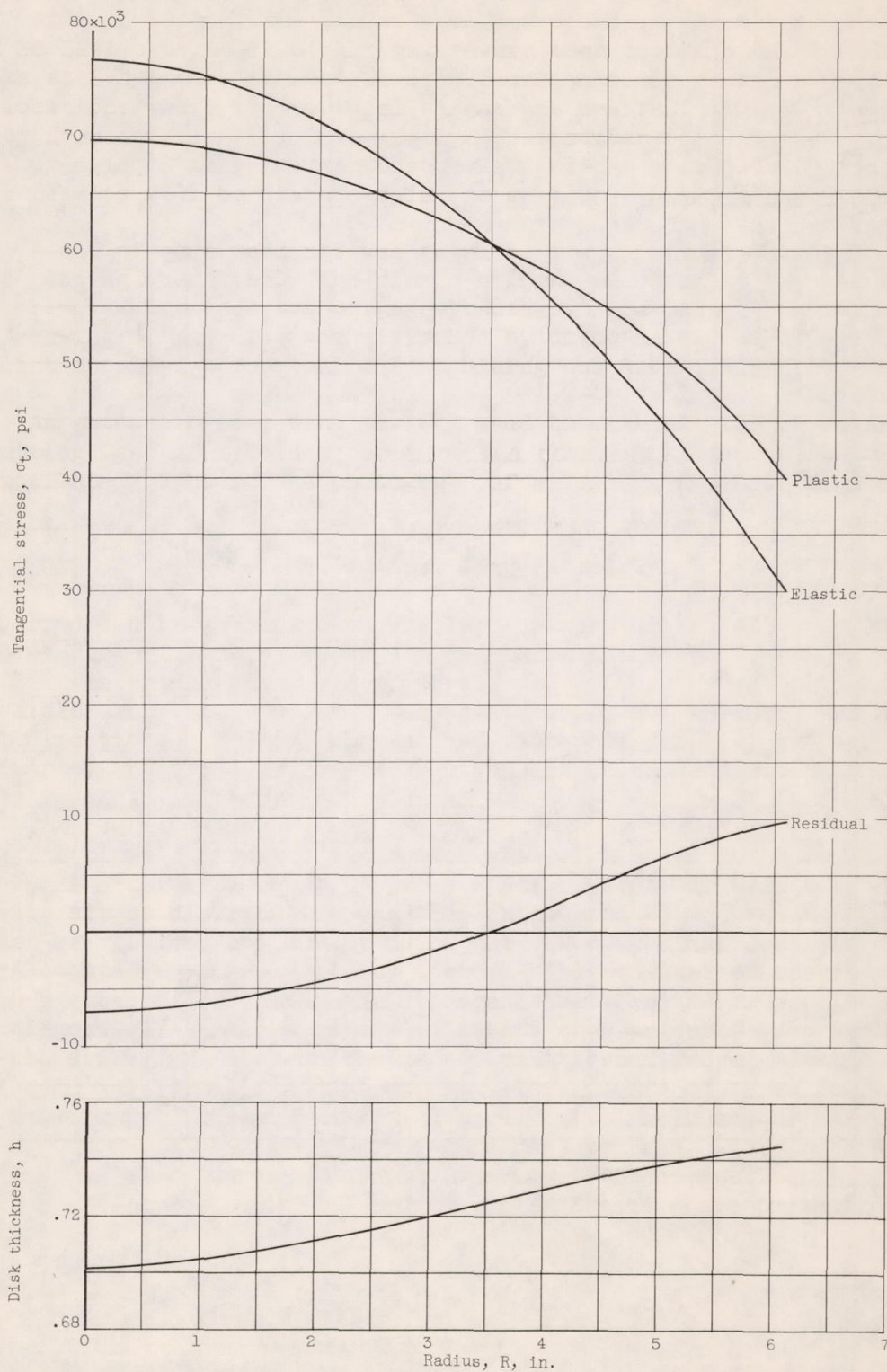


Figure 6. - Calculated plastic, elastic, and residual tangential stresses and final thickness for an overspeeded disk.

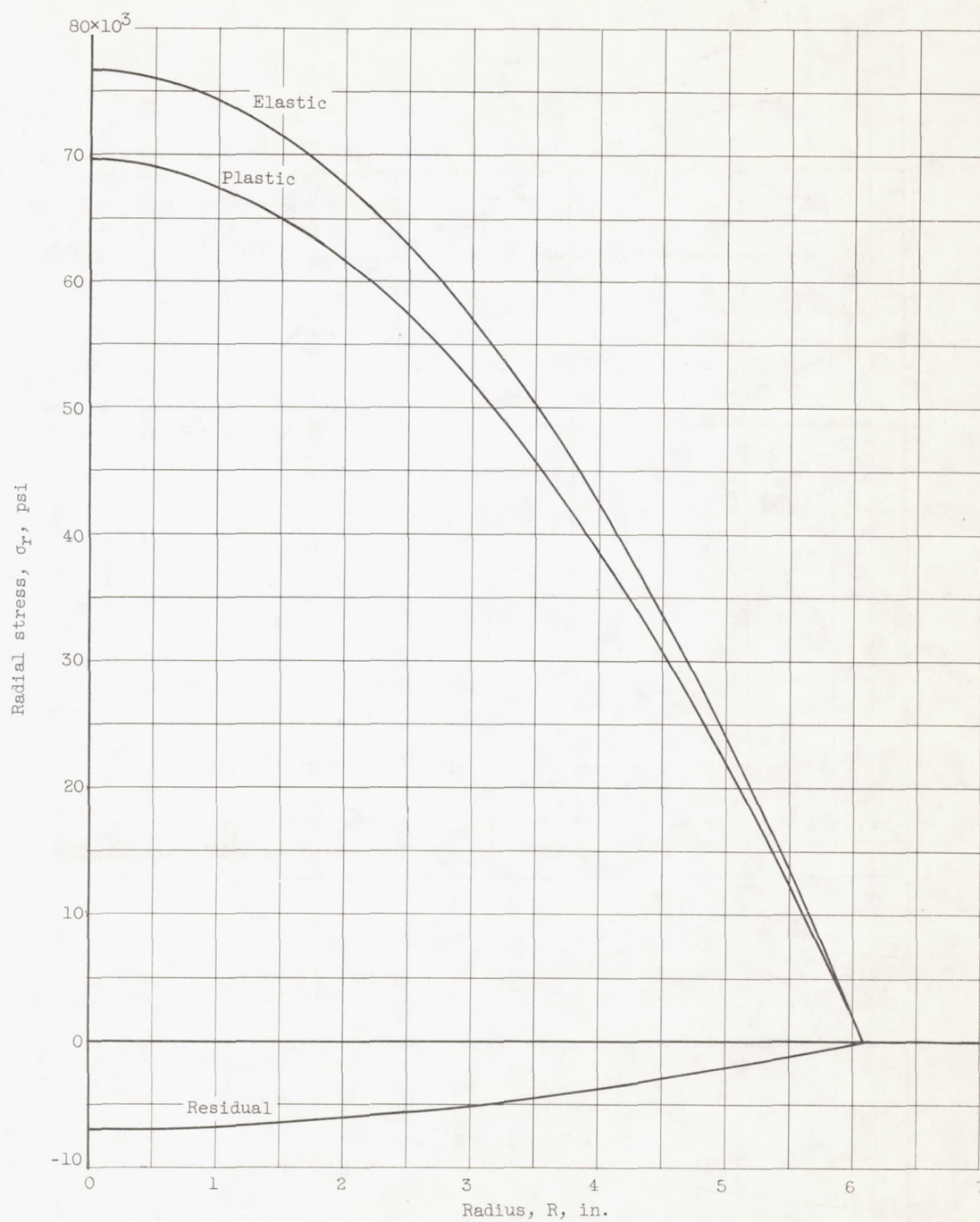


Figure 7. - Calculated plastic, elastic, and residual radial stresses for an overspeeded disk.



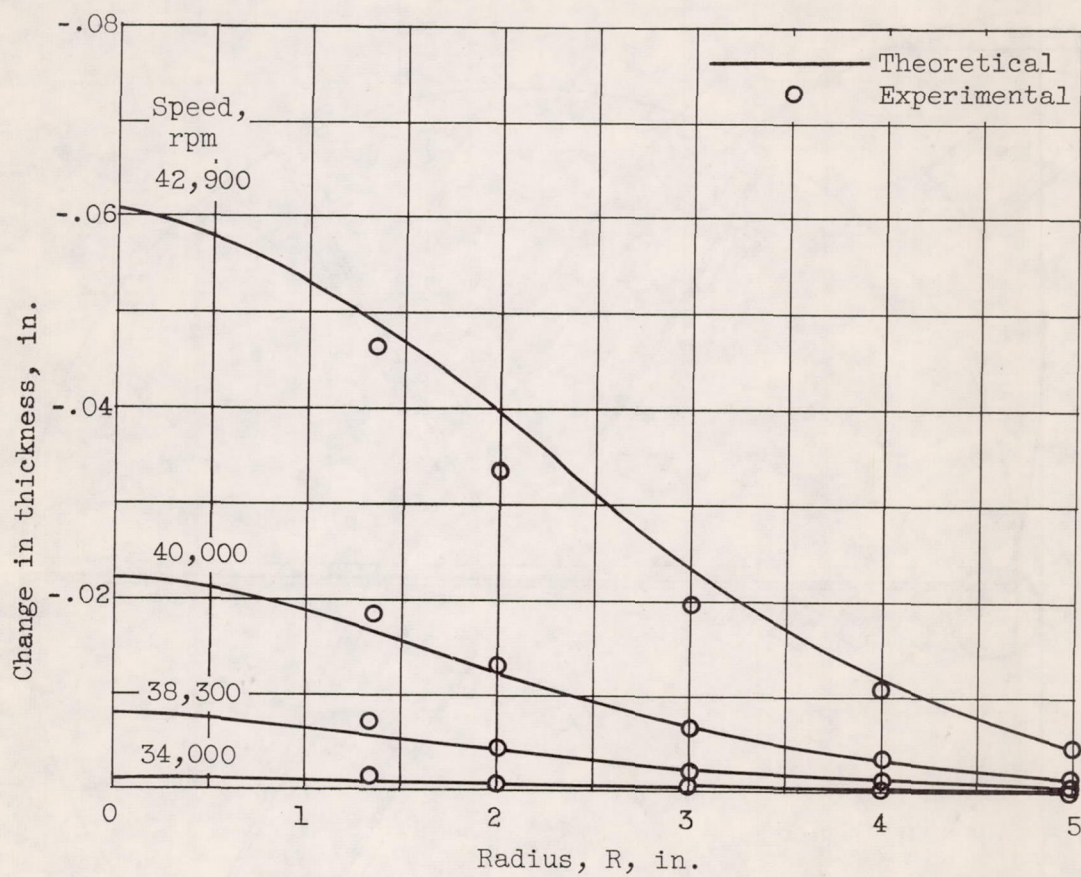


Figure 8. - Permanent change in thickness for disk tested at room temperature (ref. 4).

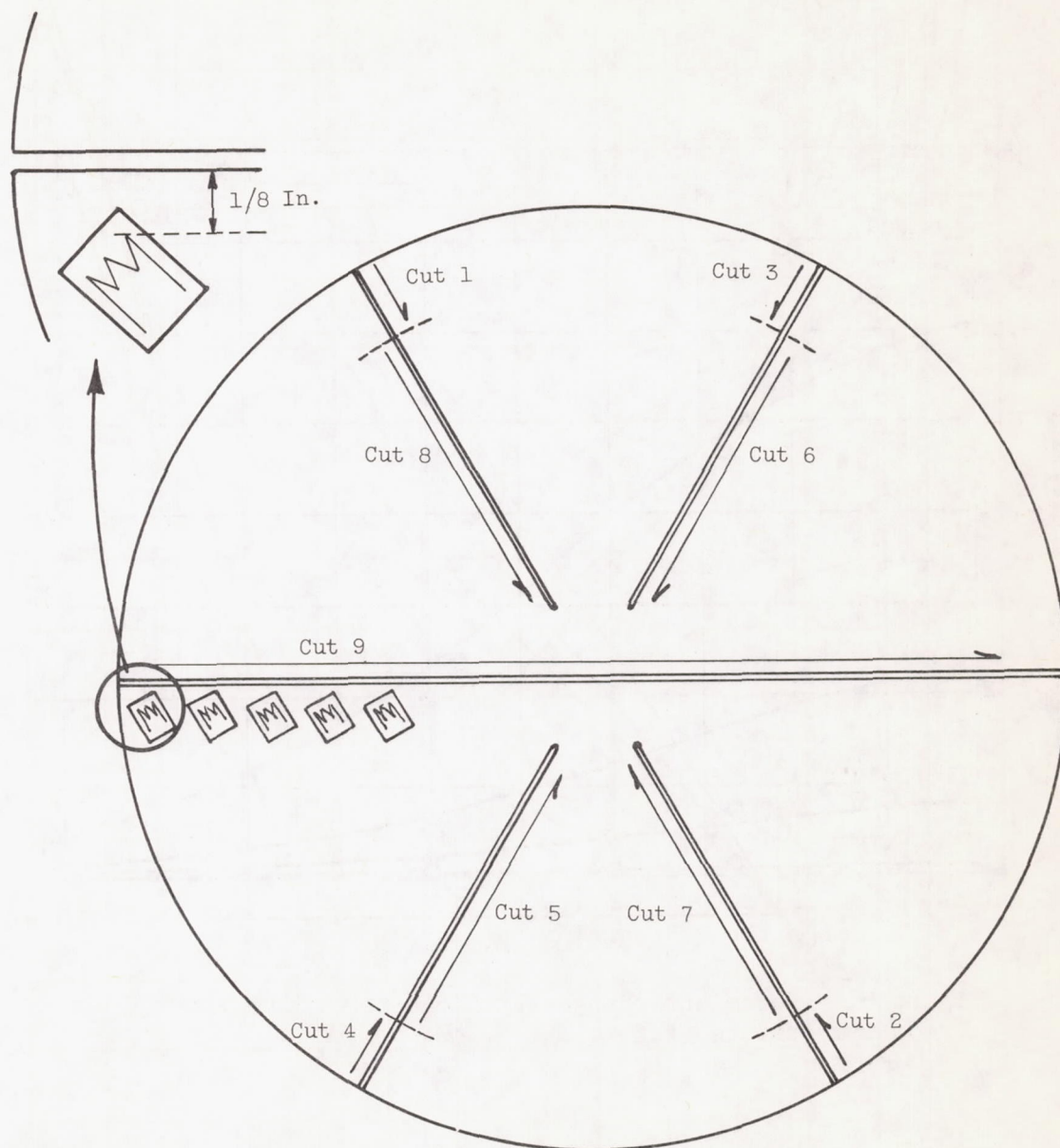
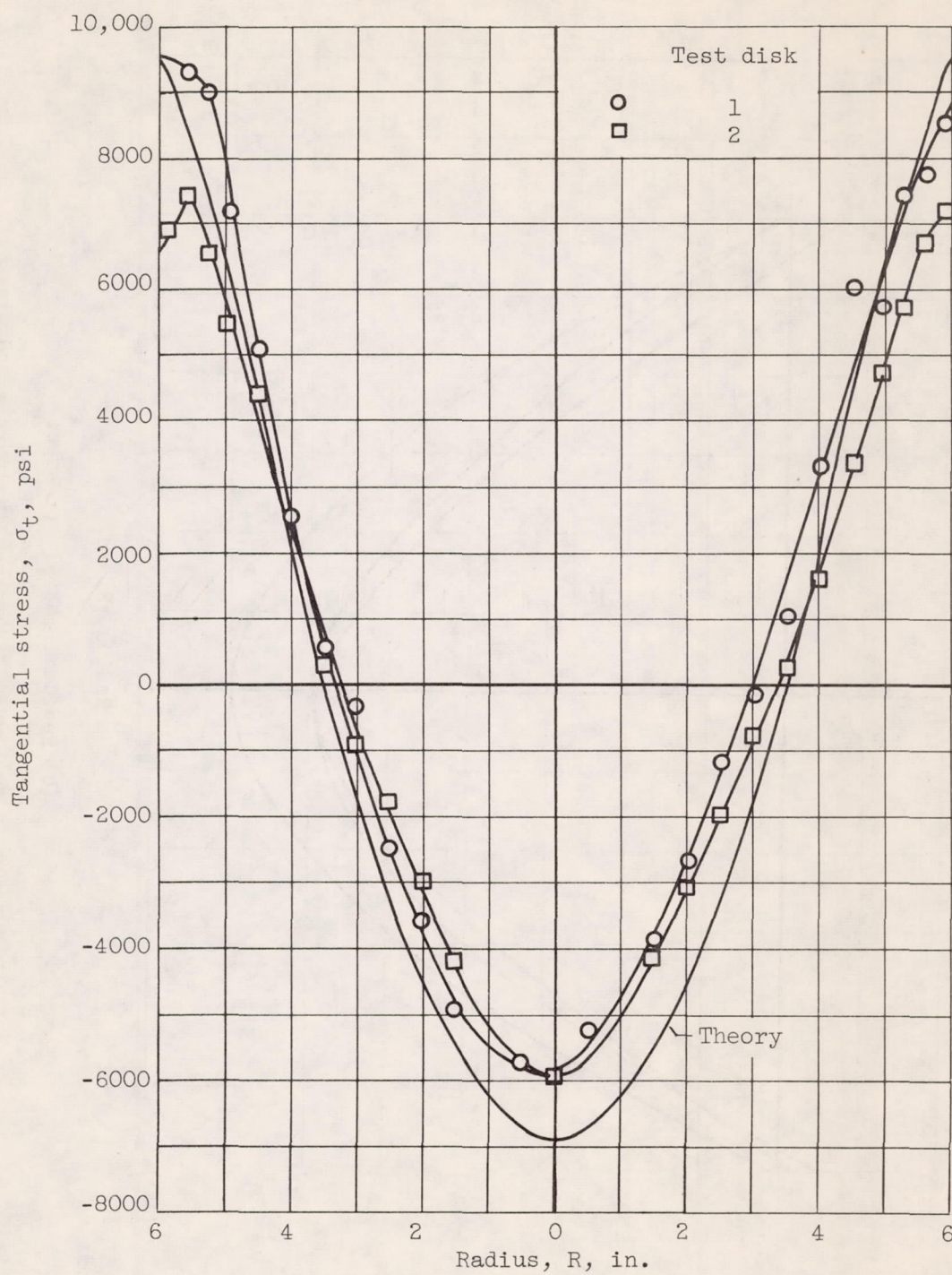


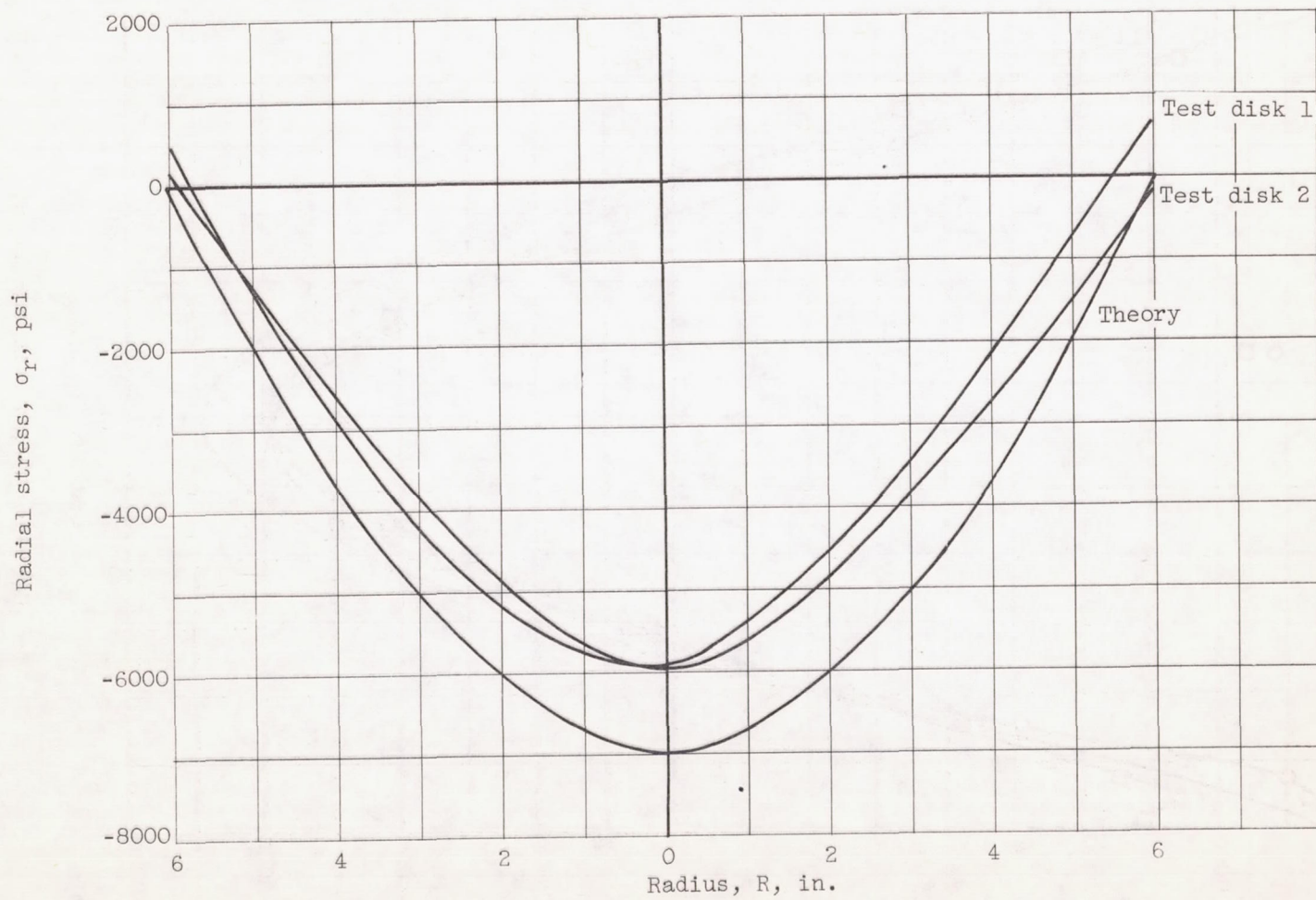
Figure 9. - Cutting procedure for relieving residual stresses in a disk.





(a) Residual tangential stresses.

Figure 10. - Stresses in overspeeded disk.



(b) Residual radial stresses.

Figure 10. - Concluded. Stresses in overspeeded disk.